



Cite this article: Bettencourt LMA, Lobo J. 2016 Urban scaling in Europe. *J. R. Soc. Interface* **13**: 20160005.
<http://dx.doi.org/10.1098/rsif.2016.0005>

Received: 3 January 2015

Accepted: 22 February 2016

Subject Category:

Life Sciences—Mathematics interface

Subject Areas:

mathematical physics, environmental science, biophysics

Keywords:

agglomeration effects, gross domestic product, urbanized area, innovation, population-size distribution, megacities

Author for correspondence:

Luís M. A. Bettencourt
 e-mail: bettencourt@santafe.edu

Electronic supplementary material is available at <http://dx.doi.org/10.1098/rsif.2016.0005> or via <http://rsif.royalsocietypublishing.org>.

Luís M. A. Bettencourt¹ and José Lobo²

¹Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

²School of Sustainability, Arizona State University, 800 Cady Mall, Tempe, AZ 85281, USA

Over the last few decades, in disciplines as diverse as economics, geography and complex systems, a perspective has arisen proposing that many properties of cities are quantitatively predictable due to agglomeration or scaling effects. Using new harmonized definitions for functional urban areas, we examine to what extent these ideas apply to European cities. We show that while most large urban systems in Western Europe (France, Germany, Italy, Spain, UK) approximately agree with theoretical expectations, the small number of cities in each nation and their natural variability preclude drawing strong conclusions. We demonstrate how this problem can be overcome so that cities from different urban systems can be pooled together to construct larger data-sets. This leads to a simple statistical procedure to identify urban scaling relations, which then clearly emerge as a property of European cities. We compare the predictions of urban scaling to Zipf's law for the size distribution of cities and show that while the former holds well the latter is a poor descriptor of European cities. We conclude with scenarios for the size and properties of future pan-European megacities and their implications for the economic productivity, technological sophistication and regional inequalities of an integrated European urban system.

1. Introduction

European nations are some of the oldest extant urban systems in the world [1–3]. Many contemporary European cities have centuries, if not millennia, of history, often stretching back to medieval or classical times. Over this long span of time, each European city has experienced periods of profound crisis alternating with booming development and has seen enormous demographic, economic, political and spatial transformations [4]. From this rich historical perspective, we may expect each European city to be exceptional and unique, and not to conform to any particular quantitative expectation [4,5].

However, the opposite perspective—that all cities share certain predictable quantitative properties—has slowly emerged from empirical studies and theoretical considerations developed by a variety of disciplines, including economics [6–8], geography [9–11], engineering [12] and complex systems [10,13–15]. All these disciplines explain the existence and development of cities as the result of the interplay between centripetal and centrifugal ‘forces’, which in turn result from socio-economic advantages of concentrating human populations in space and account for associated costs. These are known as *agglomeration* or *scaling* effects and constitute the foundational concepts for explaining the formation and persistence of cities anywhere [7,15,16]. Urban agglomeration effects are based on the observation of systematic changes in average socio-economic performance, land-use patterns and infrastructure characteristics of all cities as functions of their size. Such relations are known across the sciences as *scaling relations* [17], which relate macroscopic properties of a system—here a city—to its scale (size). For this reason, the systematic study of such relationships in cities is known as *urban scaling*.

Clearly, these two perspectives—emphasizing what is particular and what is general about cities—are at odds with each other [6,18]. Each, on its own, is too simple to be fully correct, while both should be expected to play a role to a greater or lesser extent in explaining the observed properties of any particular city. Thus, the interesting question is to what extent can the properties of any city be predicted

by general considerations and how to quantitatively assess the exceptionality of each place [18]. There is probably no better place to engage in this exercise than in Europe. Here, we tackle this tension by analysing extensive evidence for the cities of the European Union (EU) where strong national context also plays an important role on top of city-specific factors.

The empirics and the theory of urban scaling are now mature enough that quantitative expectations for scaling relations can be formulated and measured in many urban systems around the world. However, the properties of contemporary European urban systems have been studied less than those of other nations, especially the USA [6,8,11,19]. Given the movement in Europe towards greater political and economic integration, especially within the framework of the EU, it is particularly interesting to compare and contrast persistent regional differences and continental convergence among European cities.

Comparative quantitative studies of the properties of European cities have been hampered by a lack of data for consistently defined socio-economic units of analysis. General theoretical considerations and empirical practice lead us to view cities as integrated socio-economic networks of interactions embedded in physical space [8,15]. Capturing this logic when delineating urban units of analyses requires that data be collected in a consistent manner across a number of multi-dimensional criteria leading to the concept of *functional cities*. The definition of functional cities, as *integrated socio-economic units*, has become the gold standard for any scientific analysis of the properties of cities and urban systems. The US Census Bureau has a long-standing, and arguably the most consistent, definition of functional cities, known as *Metropolitan Statistical Areas* (MSAs), dating back to the 1950s and updated annually.¹ MSAs consist of a core county or counties in which lies an incorporated city (a politico-administrative entity) with a population of at least 50 000 people, plus adjacent counties having a high degree of social and economic integrations with the core counties as measured through commuting ties. MSAs are in effect unified labour markets reflecting the frequent flow of goods, labour and information, which in turn is a proxy for intense socio-economic interactions [20].

In Europe, the identification of consistent (functional) territorial units has been a goal for some time now as such definitions, and the socio-economic characteristics of the delineated territories, play important roles in the formulation of EU policies and the allocation of EU funds, for example, the structural funds for regional development and cohesion. Until recently, several systems of territorial units have coexisted in European statistical bureaus. Most are based on the Eurostat's Nomenclature of Territorial Units for Statistics (NUTS) classification system,² with urban NUTS3 corresponding roughly to integrated territorial units that can have an urban character. For these reasons, urban NUTS3 and other definitions have been the focus of several studies of agglomeration effects in European cities, using econometric analyses [16,21–25]. A unification of NUTS3 into larger functional cities has also been proposed and resulted in larger urban units (LUZ) and metropolitan areas (MAs), used in different EU statistics' urban audits. However, the NUTS system borrows heavily from underlying older, country specific, territorial units and, as such, is not consistently defined across different European nations.

An effort to define functional cities in a conceptually meaningful and empirically consistent manner has been

recently undertaken by the Organization for Economic Cooperation and Development (OECD), in collaboration with the EU [26]. This has resulted in a new set of harmonized MA definitions across the EU and other OECD nations.³ At present, these definitions represent the most consistent attempt to define functional urban areas in Europe, making contact with those of other nations such as, for example, the USA, Mexico and Japan. The advent of this dataset presents a novel opportunity to comparatively analyse the properties of European cities as a function of their population size, for which there are a number of theoretical expectations and comparative empirical evidence from other urban systems [13,15,27]. Here, we take a first step in this direction, by analysing and discussing the scaling properties of OECD–EU MAs for the five largest urban systems in Western Europe, namely France, Germany, Italy, Spain and the UK. This allows us to consider some of the properties of these national urban systems and comment on special cases and statistical uncertainties resulting from the relatively small number of large cities in each of these nations. To tackle this problem, we show how data for most cities in the EU can be pooled together while respecting national differences in social and economic development. In this way, we test urban scaling at the continental level, thus bypassing some of the statistical difficulties of small datasets in each nation. This procedure also allows us to characterize regional and national differences in urban population sizes and economic performance across different European urban systems and discuss such results in the context of the pan-European population-size distribution of cities.

2. Results

2.1. Quantitative expectations from urban scaling

We start by explicitly stating the quantitative expectations and realm of applicability of urban scaling theory as a model for analysing the empirical properties of European cities. More details about the theory and a full derivation of quantitative predictions for parameters are given in [15]. Scaling relations are naturally written in terms of scale-free functions (power-laws) [17]. Other proposals, using logarithms [28,29], also fit the data well in the regime where these functions agree analytically,⁴ but implicitly introduce a scale at which the properties of cities would have to change drastically [14]. Thus, urban scaling proposes that any city-wide property (e.g. total gross domestic product (GDP) or urbanized area), Y , should be written as

$$Y(N, t) = Y_0(t)N(t)^\beta e^{\xi(t)}, \quad (2.1)$$

where $N(t)$ denotes a city's population at time t , $Y_0(t)$ is a baseline *prefactor* common to all cities, β is a dimensionless scaling exponent (or elasticity, in the language of economics) and $\xi(t)$ are statistical fluctuations. $Y_0(t)$ is a function of time, t , capturing nation-wide socio-economic development (or decline). The exponent, β , has a special status as it is *assumed* to be time-independent, and as such a conserved quantity across time in any urban system. Theoretical considerations described below show that the exponent β is determined by general geometric considerations [15], and thus that such an assumption may be justified. Scaling analysis of ancient settlement systems lends some additional empirical support to this idea [30,31]. The variables $\xi(t)$ account for

deviations in each city from the expected (power-law) scaling relationship. As written, equation (2.1) is *exact* as any deviation from the power-law function in each city is absorbed into the corresponding ξ . Thus, the appropriateness of any scaling function to describe urban properties is tied to the statistics of ξ [14,32].

In most urban systems thus far analysed empirically, it has been found that the statistics of ξ are approximately Gaussian, with a mean over all cities in the system equal to zero ($\langle \xi \rangle = 0$) and a quantity-dependent variance, typically smaller than unity ($\sigma_\xi^2 \leq O(1)$). The properties of the variance remain largely unexplored and require further study. The properties of ξ as an approximate Gaussian random variable with zero mean justify using the simplest fitting procedure for Y versus N , a linear relation in logarithmic variables and minimizing ordinary least squares (OLSs):

$$\ln Y_i = \ln Y_0 + \beta \ln N_i + \xi_i, \quad (2.2)$$

where i indexes different cities in the urban system. This implies that the exponent β is the slope of the linear regression of $\ln Y_i$ on $\ln N_i$ and the prefactor, $\ln Y_0$, is its ordinate at the origin ($N = 1$). This also means that the scaling relation $Y(N) = Y_0 N^\beta$ is the *expectation value* of the approximately lognormally distributed stochastic variable, Y , for a city, given its population size, N , i.e. $\langle Y \rangle_N = Y_0 N^\beta$ [32].

The decomposition of any urban observable into two components, an expected value as a function of city size (scaling relation) and a local deviation ξ , parametrizes the tension between what is general and what is particular, respectively, about each city within an urban system. Because of these properties, the values of ξ_i have been used as *scale independent urban indicators* (SAMIs) to characterize the relative performance of cities within an urban system [18,33] (see the electronic supplementary material). If the dispersion (the variance, σ_ξ^2) is larger for a given urban system or a specific quantity, then the scaling relation (average expectation) is less predictive and vice versa. We will see some examples of both situations below.

Empirical analyses of the scaling relations for many urban systems have suggested that there are quantitatively consistent agglomeration effects [13,15,21,32,34] across city size in many urban systems, including Germany, China, Japan, the USA and Brazil. This translates into quantitative expectations for the exponents, β , for different urban quantities. To calculate the expected value of these exponents, urban scaling theory proposes a self-consistent model of socio-economic networks embedded in urban built space, as decentralized infrastructure networks [15]. To achieve this, it builds on a long history of earlier quantitative models [7,35,36] to describe a city functionally as a (short-term) spatial equilibrium whose spatial extent is set by the balance of density-dependent socio-economic interactions (centripetal forces) and transportation costs (centrifugal forces) [15]. The general features of this equilibrium can be obtained, as in the Alonso model [7,35,36], via a very simple argument by equating expected mobility costs, C , to the *per capita* net benefits accruing from socio-economic interactions in the city, y . The former are set by the typical length scale of the city, $L = \sqrt{A}$, where A is the area of the city, via a fractal dimension, H , of movement and a cost per unit length, ϵ , leading to $C = \epsilon A^{H/2}$. It can also be shown that socio-economic interactions are set, on average, by the population density over the built area, and thus can be written as

$y = GN/A$, where G is a constant translating interactions into urban outputs, such as the value of economic transactions [15]. Equating these costs and benefits defines the spatial extent of the city in terms of its population size, $A(N) = (G/\epsilon)^{2/(H+2)} N^{2/(H+2)}$, and consequently, the total socio-economic output of the city is $Y = yN \sim N^{1+H/(H+2)}$. This simple argument shows how treating the city as a 'bound-state' for each individual determines its spatial extent. It derives how the urbanized area of the city is naturally sub-linear on its population size, while socio-economic outputs resulting from interactions are superlinear. This effect is a spatially embedded version of 'Metcalf's law', which states that the value of a network is proportional to its number of links, not nodes [37]. While Metcalf's law is the simplest model of network effects, proposed for a simple all-to-all telecommunications network, a city is characterized by more limited increases in *per capita* connectivity with population size due to transportation costs and human effort limitations [15].

Because the calculation given so far does not consider any structure in the built space of cities, we call it the *amorphous settlement model*, which appears to describe well the area–population relationship for small historic settlements [30,31]. To obtain the exponents, β , that characterize modern cities, one needs to elaborate on the amorphous settlement model and consider a city as a spatial network of streets and infrastructure connecting places. The gradual extension of the city's networks and places creates a density-dependent growth process that alters the exponents [15], resulting in

$$\begin{aligned} \beta_{\text{socio-economic}} &= 1 + \delta, & \beta_{\text{built-infrastructure}} &= 1 - \delta, \\ \beta_{\text{employment}} &= 1, & \delta &= \frac{H}{2(H+2)}. \end{aligned} \quad (2.3)$$

The fractal dimension of movement, $2 \geq H \geq 0$, describes how individuals explore urban space. As it vanishes, $H \rightarrow 0$, individuals experience cities only from their circumscribed location, social interactions cease and the city becomes spatially segregated. As a consequence, all agglomeration effects vanish, $\delta \rightarrow 0$. In this limit, population densities or economic performance are independent of city size and indeed the (dis)advantages of urban life disappear. Thus, as $H \rightarrow 0$ cities should cease to exist, as the forces that hold them together vanish. Conversely, as $H \rightarrow 2$, individuals use the entire space of the city, which may be appropriate to describe central areas, but not the city as a whole. As $H \geq 1$ the city is fully mixing, which can be achieved at minimal movement costs for $H = 1$. Thus, $H \simeq 1$ is hypothesized to be the most likely exponent [15], corresponding to the simplest scenario with

$$\begin{aligned} \beta_{\text{socio-economic}} &= \frac{7}{6}, & \beta_{\text{built-infrastructure}} &= \frac{5}{6}, \\ \beta_{\text{employment}} &= 1, & \delta &= \frac{1}{6}. \end{aligned} \quad (2.4)$$

Urban scaling theory, as well as all other models of economic geography [7,35,36], emphasize the critical importance of using a functional definition of cities for empirical examinations of urban scaling: it is only for urban units of analysis that embody this global spatial equilibrium that the values of β are predicted via urban scaling theory and may be expected to be consistent. For other plausible urban units, such as spatial clusters of a chosen density, political or administrative cities of various kinds (e.g. municipalities, counties, etc.), there is at present, to the best of our

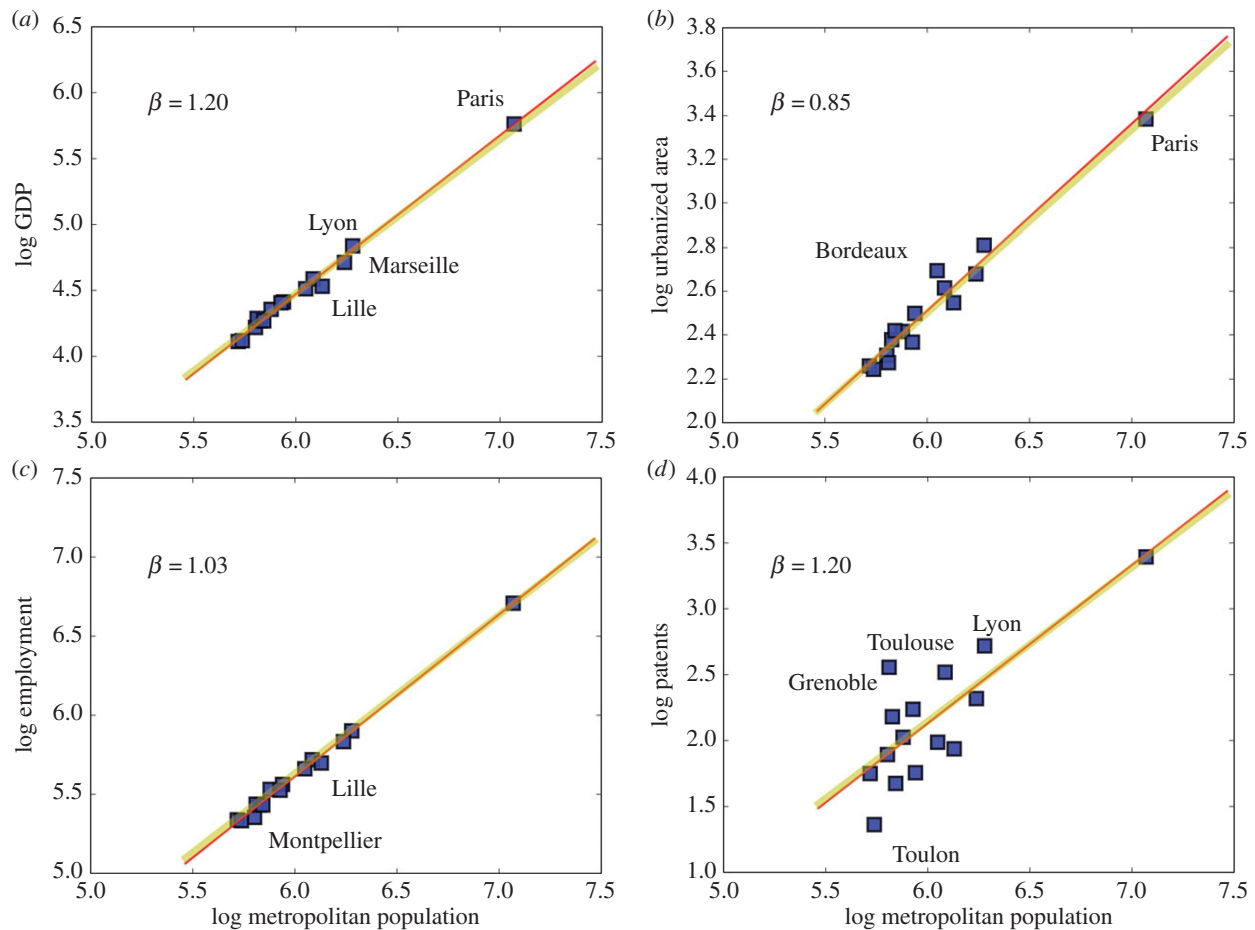


Figure 1. The scaling of urban quantities with population size for MAs in France. There are 15 functional urban areas in France with populations above 500 000 people, specifically Paris, Lyon, Marseille, Toulouse, Strasbourg, Bordeaux, Nantes, Lille, Montpellier, Saint-Etienne, Rennes, Grenoble, Toulon, Nice and Rouen. Panel (a) shows the results for GDP, with clear superlinear $\beta > 1$ scaling. Lines show the best fit (red, see table 1, $R^2 = 0.98$) and the simplest prediction from urban scaling theory (yellow). Panel (b) shows the scaling of urbanized area ($R^2 = 0.92$), (c) the scaling of total employment ($R^2 = 0.99$) and (d) of patents ($R^2 = 0.43$), as a proxy for general rates of urban innovation. The results (red lines) are statistically indistinguishable from the predictions of urban scaling (yellow lines) within confidence intervals, but the precise value of scaling exponents is hard to ascertain because of the small sample size and the level of individual city variation. (Online version in colour.)

knowledge, no theory that generates predictions for the corresponding values of β . As a result, empirical studies using various such definitions may find variable and inconsistent results [38].

The expectation of exponents with $\delta = 1/6$ from urban scaling theory is so simple that it can only be expected to hold approximately: some level of spatial, social and economic segregation always exists, and so does the opportunity to visit the city more or less extensively, especially if more accessible transportation options are available. Nevertheless, if such effects are not systematically city size dependent, they will only affect the fluctuations ξ_i for each city. Thus, we will use these values of β as null models in our empirical analyses and find below that these expectations hold surprisingly well for modern Europe, especially in the aggregate.

2.2. Urban scaling analysis in five European nations

We proceed by analysing the general properties of European MAs for the largest five urban system in Western Europe. These are some of the oldest urban systems in the world. All five urban system have long roots in history, dating back to Roman times in some cases (Italy, France, parts of Britain) and the medieval period for most (i.e. Germany),

and persisting through much change and transformation [4,5]. The UK's urban system was the first in the world to undergo the industrial revolution with well-known consequences for the growth of its cities and the change in the living conditions of its inhabitants [4,5]. France and Germany followed suit shortly thereafter. In addition, these five urban systems have experienced very different levels of political and economic unification, with Italy and Germany being unified relatively recently and Germany being subsequently separated into East and West at the end of World War II. Finally, over the last few decades all these nations have become integrated as part of the EU and granted free circulation of people (citizens) and capital. For all these reasons, we may expect all five different urban systems to exhibit different properties.

2.2.1. France

France has one of the oldest politically and economically integrated urban systems in Europe [39,40]. Figure 1 shows the scaling behaviour of all 15 cities in France with population above 500 000 people, for urban GDP, urbanized area, employment and patents.

Urban scaling theory predicts superlinear behaviour $\beta > 1$ for socio-economic quantities (GDP, patents), linear behaviour

Table 1. Summary scaling exponents for GDP, urbanized area, employment and patents for European MAs versus population. See text and figures 1–7, electronic supplementary material, figures S1–S8 for additional details and electronic supplementary material for labour productivity, unemployment and CO₂ emissions. Square brackets show 95% CIs on exponents.

nation	N_c	GDP	urbanized area	employment	patents
France	15	1.20 [1.15,1.26]	0.85 [0.75,0.95]	1.03 [0.98,1.07]	1.20 [0.72,1.69]
UK	15	1.12 [1.00,1.25]	0.90 [0.85,0.95]	1.00 [0.97,1.02]	n.a.
Spain	8	1.13 [0.97,1.30]	1.09 [0.86,1.32]	1.07 [0.99,1.16]	1.66 [0.95,2.37]
Italy	11	1.08 [0.82,1.35]	0.86 [0.68,1.04]	1.02 [0.85,1.18]	1.07 [0.40,1.73]
Germany	24	1.17 [1.06,1.28]	0.95 [0.84,1.06]	1.02 [0.98,1.07]	1.30 [0.92,1.67]
Europe	102	1.17 [1.11,1.22]	0.93 [0.88,0.98]	1.02 [1.00,1.05]	1.13 [0.91,1.34]

for characteristics closely tied to population, such as employment ($\beta = 1$), and sublinear behaviour for urbanized area ($\beta < 1$), expressing greater average densities in larger cities. All three trends are observed for the French urban system [25] with small statistical dispersion (deviations, $\xi \neq 0$), with the exception of patents, which is always a noisier quantity [41] (figure 1*d*). The estimation results obtained using OLS regression on logarithmic variables produces scaling exponents that agree quantitatively with the simplest predictions from urban scaling theory. Unfortunately, the small sample size does not allow very precise exponent measurements and confidence intervals remain broad (but clearly super/sublinear as expected), especially for patents (table 1). We will address this issue below, after seeing the problem recur for other European urban systems.

Regarding exceptions, the cities of France are very well behaved and deviations from their average scaling relation are not strong (see the electronic supplementary material). Nevertheless, figure 1*a* for GDP reveals that cities such as Lille and Marseille have smaller economies than would have been expected for their population size (electronic supplementary material, figure S4). Most cities in France also have an extent of urbanized area that is very consistent with a nation-wide scaling trend; an exception is Bordeaux that appears larger than expected for its population (electronic supplementary material, figure S5). Figure 1*c* shows that Lille, again, and Montpellier have lower employment than expected (see the electronic supplementary material, figure S6). Finally, figure 1*d* expresses well-known qualitative expectations that technological innovation is an important feature of French cities such as Grenoble, Toulouse and even Lyon, whereas Toulon in Provence shows very little inventive activity for a French or indeed European city (electronic supplementary material, figure S7).

2.2.2. United Kingdom

Analogous to France in many ways, Great Britain is also an old and fairly politically unified urban system [42]. There are also 15 MAs in Great Britain with populations above 500 000. These cities also exhibit tight scaling behaviour with small deviations (figure 2). Exponent estimates for GDP, urbanized area and employment agree with those of France and with the expectations of urban scaling theory, but statistically suffer also from containing a small number of cities. Both French and British urban systems show

strong macrocephaly, with Paris and London (the two largest cities in the EU, with population approx. 11.5 million) being much larger than secondary cities in each nation. This also means that Paris and London manifest much stronger agglomeration (dis)advantages than any other cities in their national setting.

Some notable exceptions can nevertheless be identified. As is well documented, the former large-scale manufacturing centres of Birmingham and, to a lesser extent, Manchester (the two largest cities in Britain after London) show economies and levels of employment that are too small for their population size. Edinburgh, the capital of Scotland, on the other hand, behaves in the opposite direction and is richer than expected for a city of its size in the British or European context (figure 2*b* and electronic supplementary material, figure S4). Unfortunately, the present OECD–EU data release does not provide numbers for patents produced in British cities. This issue has been the focus of some empirical controversy [38,43]. On a separate piece, we show that British inventors file the majority of their patents in the USA, and that when this is taken into account, strong agglomeration effects are observed. It will be important to continue to understand the nature and magnitude of measures of innovation in British cities, as there is often the perception that these activities have become too concentrated in London, but see [44] and the discussion of the next section.

2.2.3. Spain

Spain is the smallest of the national urban systems analysed here, with only eight cities above 500 000 people (figure 3). Nevertheless, GDP, employment and patents scale as expected, although with wide confidence intervals (table 1). The urbanized area of Spanish cities appears superlinear, contrary to theory, though with a very wide confidence interval: this is largely the result of the urbanized area for Madrid, which is very large, even in the context of all large EU cities (electronic supplementary material, figure S5) as we shall see in greater detail below.

Among such a small number of cities the specific characteristics of individual places become particularly important. We see that some of the cities of Spanish Southwest, such as Seville and Malaga are poorer and less inventive than their national counterparts and that cities such as Barcelona and Zaragoza produce a number of patents much larger than Las Palmas (Canary Islands), even when accounting

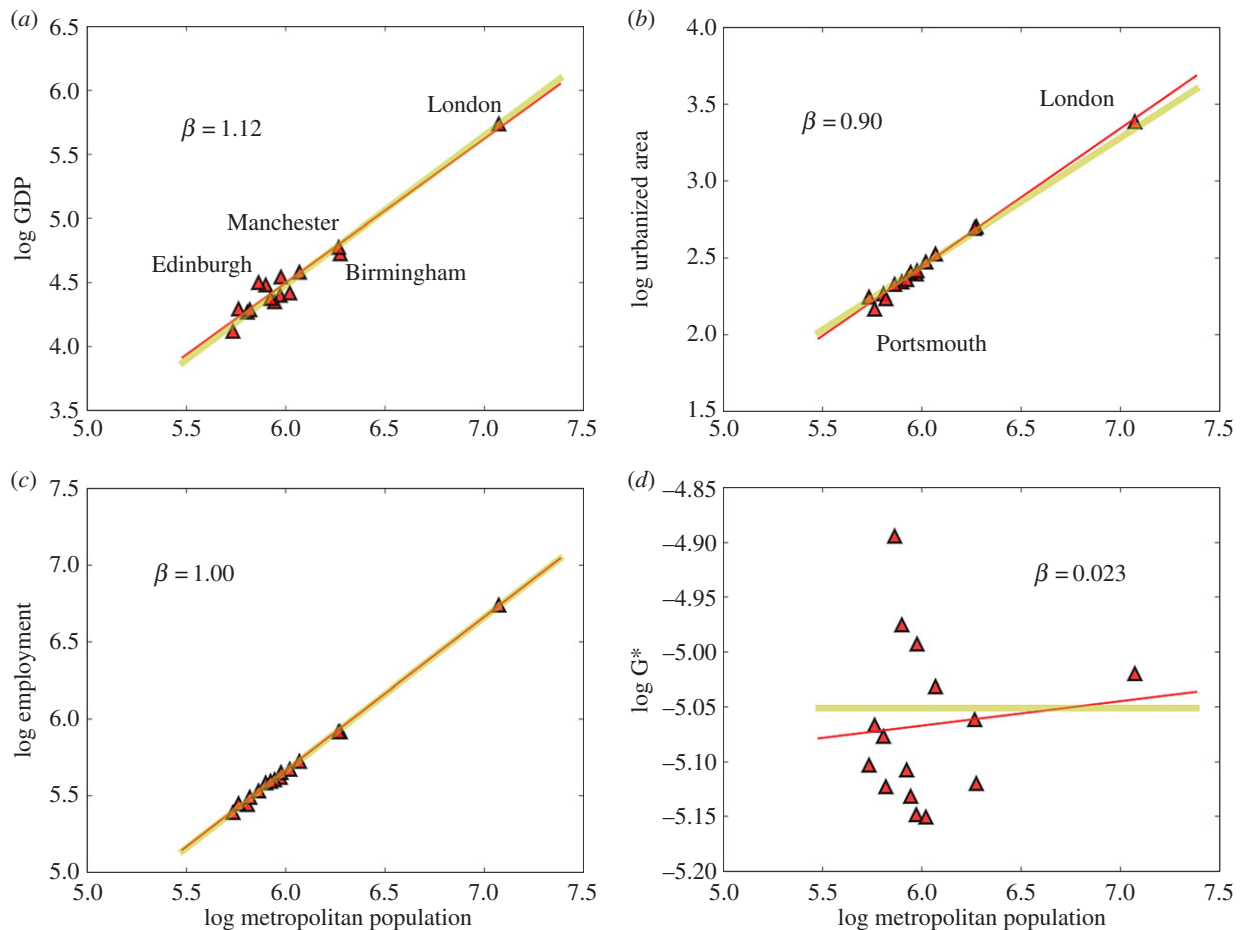


Figure 2. The scaling of urban quantities with population size for MAs in the UK. There are 15 functional cities above 500 000 people in the dataset, specifically London, Birmingham, Leeds, Bradford, Liverpool, Manchester, Cardiff, Sheffield, Bristol, Newcastle, Leicester, Portsmouth, Nottingham, Glasgow and Edinburgh. (a) Results for GDP, with clear superlinear $\beta > 1$ scaling. Lines show the best fit (red, see table 1, $R^2 = 0.92$), and the simplest prediction from urban scaling theory (yellow). (b) The scaling of urbanized area ($R^2 = 0.98$), (c) the scaling of total employment ($R^2 = 0.99$) and (d) the product of urbanized area *per capita* times GDP *per capita* ($R^2 = 0$), which is predicted by urban scaling theory to be city size invariant, as observed. We see that the best-fit results (red lines) are in broad agreement with urban scaling theory (yellow lines) and evidence from other urban systems, but that confidence intervals for parameters are wide because of the smallness of the data sample (table 1). (Online version in colour.)

for their respective population sizes (electronic supplementary material, figure S7). A small urban system such as Spain's thus allows only a very general comparison with expected agglomeration effects because large uncertainties remain as to the value of average elasticities or exponents.

2.2.4. Italy

Much Spain, but slightly larger, the urban system of Italy comprises 11 cities over 500 000 people (figure 4). The most striking feature of the Italian urban system are the differences between the northern and southern regions of the country. Results agree generally with the predictions of urban scaling theory but superlinear effects of GDP and patenting are a little lower than expectations, although with very wide 95% confidence intervals. This is partly because Naples is a strong outlier along a number of dimensions (electronic supplementary material, figures S4–S7): it has a small GDP, employment and number of patents for its population size and is also small in terms of its urbanized area. Moreover, Naples is not alone and other southern Italian cities—such as Palermo, Catania and Bari—also underperform in terms of GDP, levels of employment and patenting, not only within Italy but in the context of all other European cities (see the electronic supplementary material).

2.2.5. Germany

Finally, we turn to the scaling analysis of the German urban system, the largest of the five European nations analysed here with 24 urban areas of more than 500 000 people. Figure 5a shows the scaling of GDP for German MAs versus their population size. The best-fit line agrees perfectly with the simplest prediction of urban scaling theory (table 1) although the East–West divide between the nation is also apparent, with Berlin, and to a lesser extent Dresden and Leipzig standing out below the scaling line. These cities are also exceptionally poor for their size within the full European context, with an economic performance on par with southern Italian cities, such as Naples and Palermo (electronic supplementary material, figure S4). Figure 5b shows the scaling of urbanized area, with best-fit line sublinear but a little higher than urban scaling predicts. It is important to note that the East–West divide is also visible here with Eastern cities showing larger urbanized areas than expected (electronic supplementary material, figure S5). Employment shows a very predictable linear trend, with the exception of Bremen, which shows smaller number of jobs than expected, a well-documented phenomenon after the decline of its shipyards and other related industries [45]. Finally, the trend for patents is quite noisy, but shows well-known technology

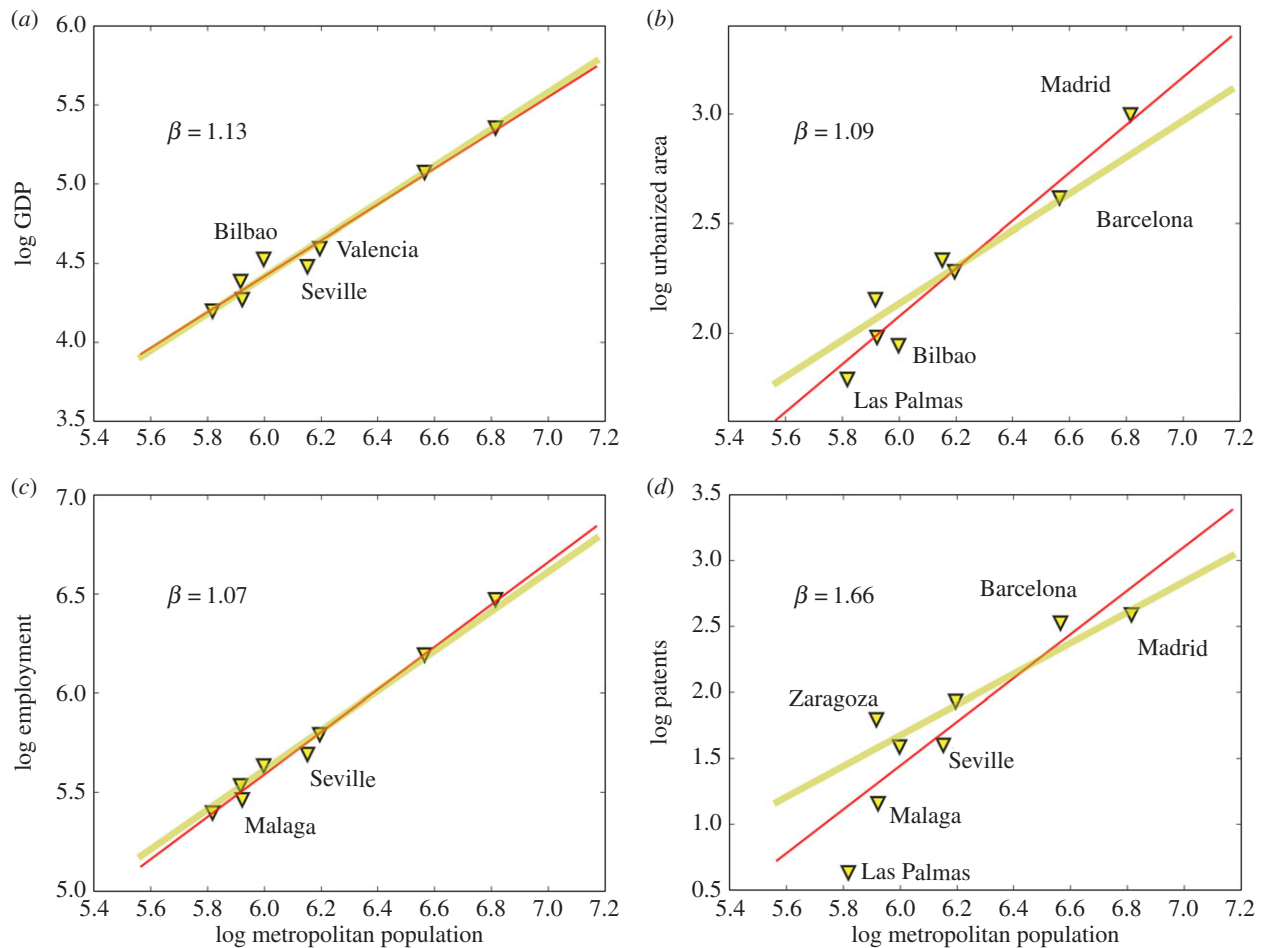


Figure 3. The scaling of urban quantities with population size for MAs in Spain. There are only eight functional cities above 500 000 people in the dataset, specifically Madrid, Barcelona, Valencia, Seville, Zaragoza, Malaga, Las Palmas and Bilbao. (a) The results for GDP, with superlinear $\beta > 1$ scaling. Lines show the best fit (red, see table 1, $R^2 = 0.94$) and the prediction from urban scaling theory (yellow). (b) The scaling of urbanized area ($R^2 = 0.88$), (c) the scaling of total employment ($R^2 = 0.98$) and (d) of patents ($R^2 = 0.62$). Because of the small number of large cities in Spain, as well as individual and regional variations, the confidence intervals on exponents are particularly broad (table 1). (Online version in colour.)

centres such as Munich, Stuttgart and Mannheim as strong positive outliers, whereas Leipzig and Bremen appear as cities with low rates of invention (electronic supplementary material, figure S7).

It is noteworthy that, despite strong regional and city-specific differences, the larger number of cities in Germany allows us to start establishing the quantitative superlinear character of GDP and patenting, the strictly linear behaviour of employment and the sublinear nature of urbanized area with more statistical confidence. The analysis of these five largest western European urban systems shows, however, that given the typical statistical dispersion in the character of each city, larger samples would be necessary to establish the actual values of scaling exponents with sufficient confidence that some testing of theory can be performed. Such a test requires therefore a larger number of European cities, an issue that we address in the next section.

2.3. The pan-European urban system

Estimating scaling parameters for relatively small urban systems, with less than a few dozen large cities, is fraught with procedural difficulties and typically leads to large error bands. This makes it difficult to assess the consistency of scaling parameters across nations and over time, and permits only relatively weak conclusions. Fortunately, the simple mathematical form of scaling relations provides us with a

general method to pool data together from across urban systems that we expect *a priori* have different baseline quantities, such as greater/smaller wealth in Germany/Spain. Besides being mathematically and econometrically justified, pooling the data from the various national systems is also conceptually interesting, given the efforts at European integration via trade and financial networks, and commonalities of legal, institutional and technological frameworks. This procedure will allow us to discuss the extent of this integration along several independent dimensions.

To see this, consider the general form of the scaling relation in logarithmic variables equation (2.2). The average of $\ln Y_i$ over all cities, $\langle \ln Y \rangle$, is

$$\langle \ln Y \rangle = \frac{1}{N_c} \sum_{i=1}^{N_c} \ln Y_i = \ln Y_0 + \beta \langle \ln N \rangle, \quad (2.5)$$

where N_c is the number of cities in a given urban system (nation) and where we have used the fact that $\langle \xi \rangle = 0$ for a well-posed fit. Subtracting equation (2.6) from equation (2.5), we obtain

$$\Delta \ln Y_i = \beta \Delta \ln N_i + \xi_i, \quad (2.6)$$

with $\Delta \ln Y_i = \ln Y_i - \langle \ln Y \rangle$ and $\Delta \ln N_i = \ln N_i - \langle \ln N \rangle$. This relationship is now a *centred* scaling relation. In logarithmic scales, it is a straight line with slope β and coordinate at the origin pinned to zero. Two different urban systems,

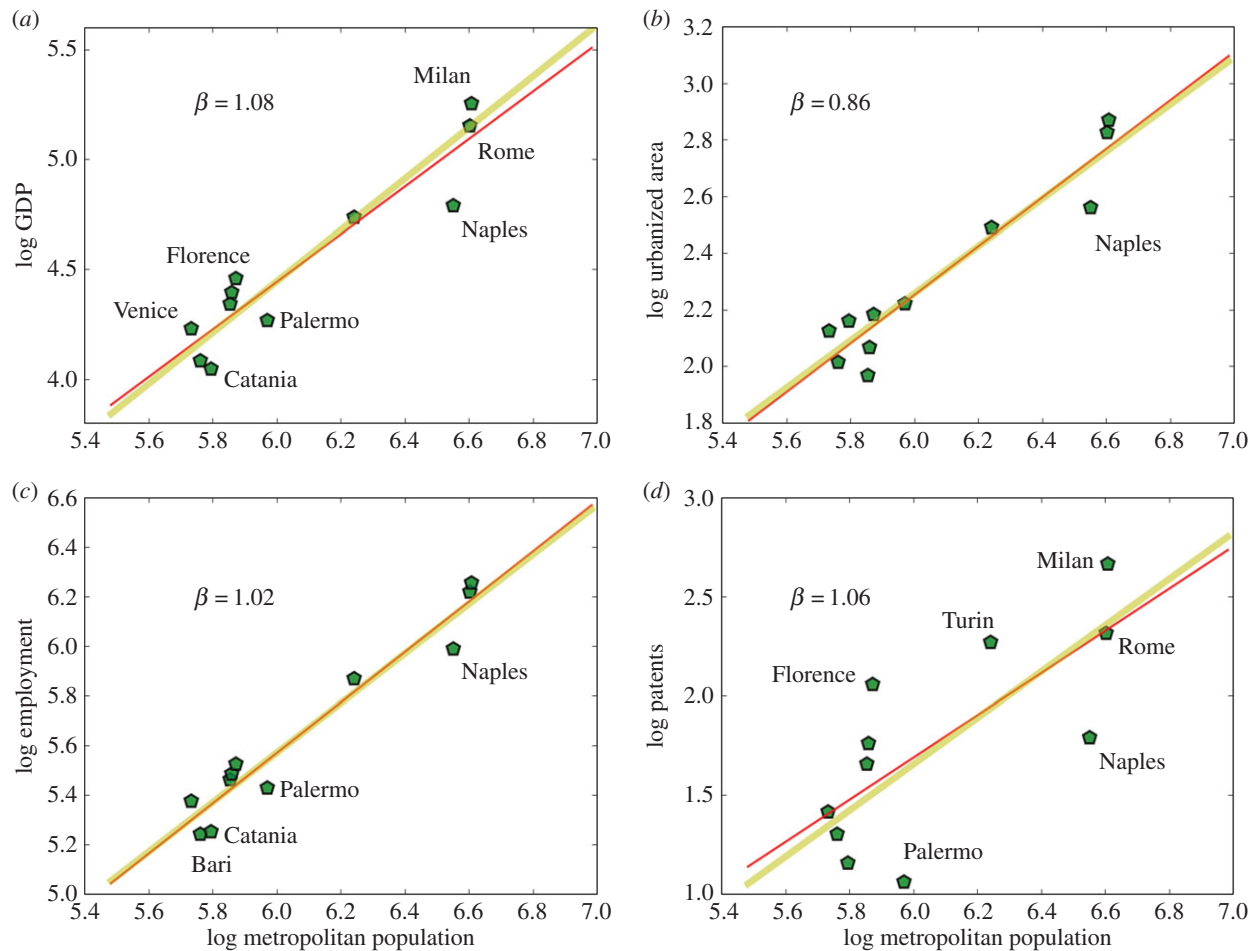


Figure 4. The scaling of urban quantities with population size for MAs in Italy. There are just 11 functional cities above 500 000 people in the dataset, namely Rome, Milan, Naples, Turin, Palermo, Genova, Florence, Bari, Bologna, Catania and Venice. (a) The results for GDP. Lines show the best fit (red, see table 1, $R^2 = 0.78$) and the prediction from urban scaling theory (yellow). (b) The scaling of urbanized area ($R^2 = 0.83$), (c) the scaling of total employment ($R^2 = 0.89$) and (d) of patents ($R^2 = 0.29$), as a proxy for general rates of urban innovation. The small number of large cities in Italy, compounded by the strong north–south divide in development makes the Italian urban system far from regular from the point of view of urban scaling, resulting in lower R^2 s than for other nations and in broad confidence intervals for estimated exponents (table 1). (Online version in colour.)

after centring, share the same origin (0, 0) in logarithmic axes and can be superposed. Thus, the centred scaling relation is a one-parameter model that can be used to determine the scaling exponent in a way that excludes covariations of the intercept and exponent during estimation.

Using this procedure, we can centre variables from different urban systems onto the same dataset and perform a global scaling analysis to estimate the overall scaling exponent β . Figure 6 shows the result of this procedure using OECD–EU MAs for 12 European nations (102 cities). This enlarged set of observations include, in addition to the urban systems analysed above, cities from Austria, Belgium, the Czech Republic, The Netherlands, Poland, Sweden and Switzerland. We excluded urban systems with two or fewer MAs, such as Portugal or Norway.

Figure 6a shows the scaling relation for GDP across Europe. We find perfect agreement between urban scaling theory and the data using a one-parameter best fit. For urbanized area, the fit diverges somewhat, but the prediction of urban scaling theory ($\beta = 5/6$, no fit) hits precisely the area of both smaller and largest cities (Paris and London), which the fit misses. Agreements for employment and patents are also excellent.

We have also performed a scaling analysis of other quantities in this way and analysed the structure of their residuals (see the electronic supplementary material, figures S1–S8).

For example, for *labour productivity*, we find a scaling exponent of $\beta = 1.16$ (95% CI [1.12, 1.20], $R^2 = 0.94$) statistically indistinguishable from the simplest prediction from urban scaling theory ($\beta = 7/6$) (see the electronic supplementary material, figure S1). For unemployment, we find an approximately linear exponent of $\beta = 1.02$ (95% confidence interval [0.95, 1.10], $R^2 = 0.77$) (electronic supplementary material, figure S2) and for CO₂ emissions a best-fit exponent of $\beta = 1.12$ (95% confidence interval [0.98, 1.26], $R^2 = 0.53$) (electronic supplementary material, figure S3). Analysis of the ranked residuals fleshes out the character of some of the exceptional cities noted above as these residuals can be used to compare urban performance independently of their size (and, after centring, of their national contexts). Naples and cities of southern Italy (Palermo, Catania) as well as Berlin and cities of eastern Germany show low economic performance (electronic supplementary material, figure S5) and high unemployment (electronic supplementary material, figures S2 and S6) even when compared across Europe. Eindhoven is the most inventive European city, followed by Grenoble, Bologna and Stuttgart, whereas Las Palmas, Naples, Palermo and Toulon produce an abnormally small number of patents (electronic supplementary material, figure S7). The pattern of residuals in CO₂ emissions is rich and diverse and invites further analysis

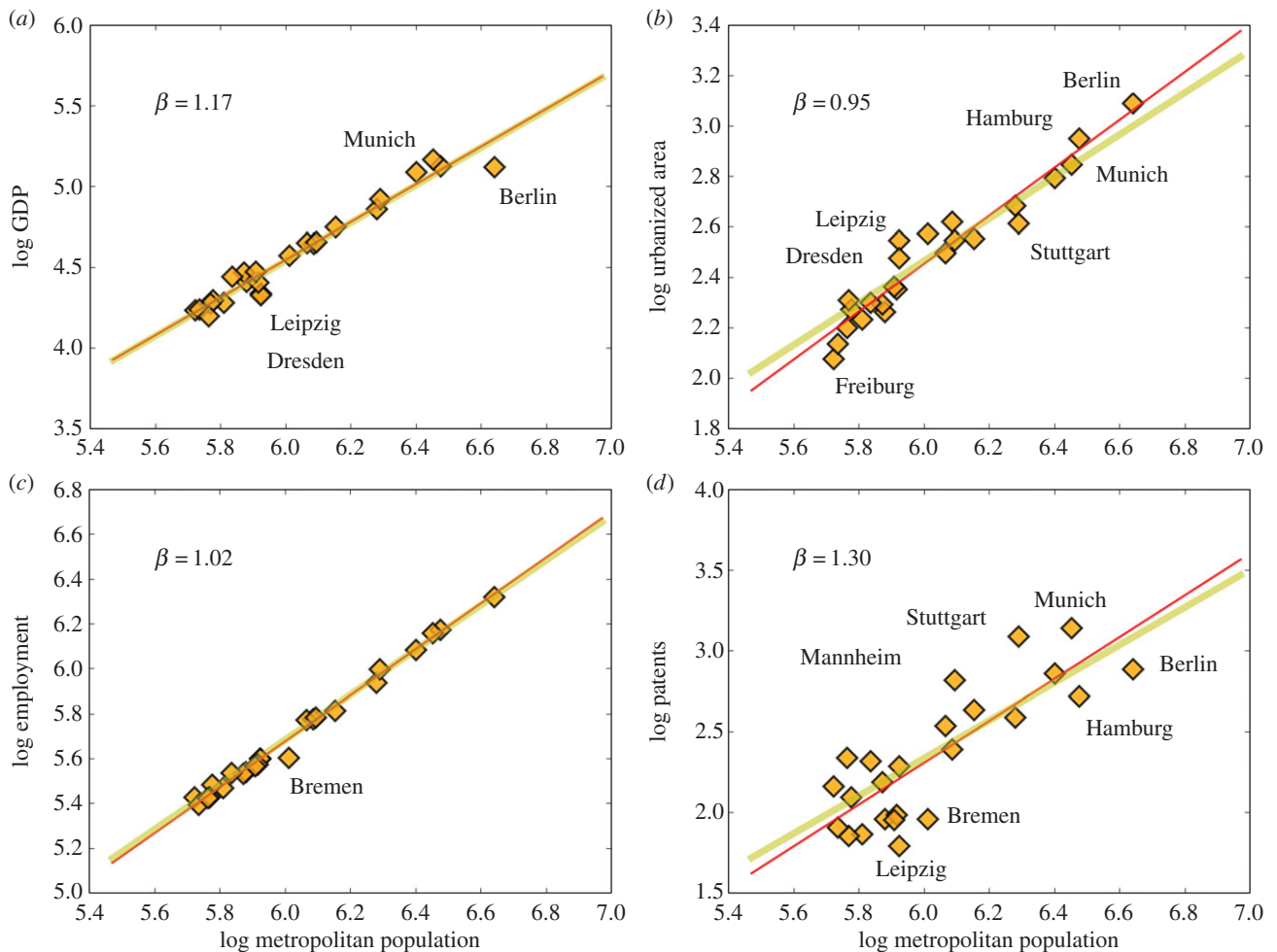


Figure 5. Urban scaling for MAs in Germany. There are 24 functional cities above 500 000 people in the dataset, specifically Berlin, Hamburg, Munich, Cologne, Frankfurt, Stuttgart, Essen, Leipzig, Dresden, Dortmund, Düsseldorf, Bremen, Hanover, Nuremberg, Bochum, Freiburg im Breisgau, Augsburg, Bonn, Karlsruhe, Saarbrücken, Duisburg, Mannheim, Münster and Aachen. (a) The results for GDP, with clear superlinear $\beta > 1$ scaling. Lines show the best fit (red, see table 1, $R^2 = 0.91$) and the prediction from urban scaling theory (yellow). (b) The scaling of urbanized area ($R^2 = 0.86$), (c) the scaling of total employment ($R^2 = 0.98$) and (d) of patents ($R^2 = 0.47$), as a proxy for general rates of urban innovation. Despite the large size of the German urban system, clear differences between East and West are still visible, especially in the behaviour of Berlin. (Online version in colour.)

and interpretation in the future (electronic supplementary material, figures S3 and S8).

We conclude that the pooled dataset for Europe shows good general agreement with urban scaling theory and that the variability observed at the national level is a consequence of small datasets and of the levels of typical variation in cities and regions. In this way, we explicitly see how urban scaling is an emergent property of functional cities that becomes visible statistically as more cities are considered.

2.4. A European city with 50 million people? City size distributions and scaling

We have just seen how data for different urban systems can be pooled together, after centring, to provide a larger sample for which urban scaling effects can be empirically tested in a very simple and robust way. In doing this, we normalized the data for each country by the average logarithmic city size ($\langle \ln N \rangle$) and indicator magnitude ($\langle \ln Y \rangle$) within the sample. Analysing the magnitude of these variables for each nation gives us a sense of their convergence or divergence within the European system. Figure 7a shows the average

logarithmic GDP and population for cities in the 12 European nations pooled in figure 6.

As expected by construction, these points show no correlation between the two variables and thus appear fairly scattered. A group of nations clusters together in the centre, including Austria, Belgium, France, Germany, Sweden and the UK. These nations and their urban systems appear more integrated than other outliers, such as Switzerland, the Czech Republic, Poland or even Spain. Tracing a vertical line of approximate same average logarithmic population, we cross, from bottom to top, France, the UK, Germany and The Netherlands. This means that with the same average city size (but different distributions, as we discuss below), these nations have cities with increasingly larger economies. In other words, the economy of The Netherlands uses its urbanization much more efficiently to produce economic value than Germany's, followed by the UK and then by France.

An analogous argument can be developed along a horizontal line, tracing nations with the same average economic performance per city but with different average city sizes. From left to right, we see, roughly along the same horizontal line, Switzerland, Austria, Sweden, Germany and Spain. A slightly lower (poorer) horizontal line may include

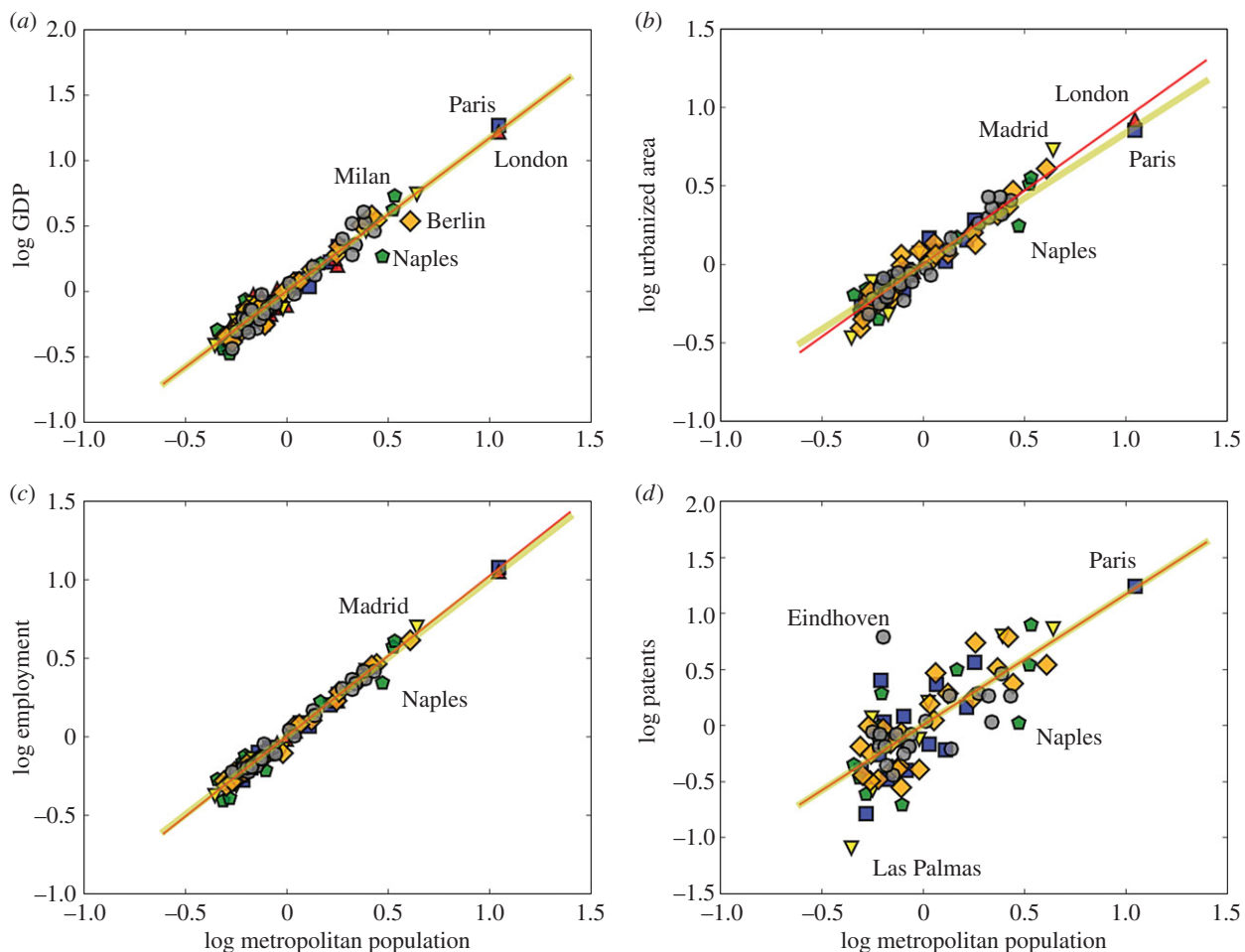


Figure 6. The scaling of urban quantities with population size for MAs in Europe. These data include all urban systems in the EU and Switzerland with more than two cities above 500 000 people (102 functional cities in 12 nations: Austria, Belgium, the Czech Republic, France, Germany, Italy, The Netherlands, Poland, Spain, Sweden, Switzerland and the UK). Because the data have been centred in each nation (see text) the expected scaling relation has intercept zero in a logarithmic plot. We show the simplest prediction of urban scaling theory as a yellow line in each panel, setting superlinear exponents at $\beta = 7/6$ and sublinear at $\beta = 5/6$. With these choices, there are no free parameters and a direct test of urban scaling can be performed without statistical uncertainties arising from best fits in small data samples. (a) Shows the results for GDP and a nearly exact agreement with theory (best fit, $R^2 = 0.90$; table 1). (b) Shows the results for urbanized area, the best fit gives a slightly larger β than predicted by theory (red line, $R^2 = 0.88$), but fails to describe the largest cities. Urban scaling fits most of the data well and correctly predicts the urbanized area of London and Paris. Employment (c) is also linear as expected ($R^2 = 0.97$). Finally, patents in (d) are noisier, but the best fit (red line, $R^2 = 0.30$) and prediction from urban scaling theory (yellow line) are statistically consistent and, in particular, predict well innovation rates for Paris. Data for patents are not provided for the UK and are inconsistent for Poland, so that these two nations are not included in the analysis of figure 1d. (Online version in colour.)

Belgium, the UK and Italy. This shows that Switzerland requires smaller cities to achieve the same economic performance of, say, Germany, and that Spain is able to belong to this club by having larger cities, that is, by further exploring the economic magnification effects of superlinear scaling.

Thus, the path to a richer nation overall depends on two important but uncorrelated dynamics: baseline productivity per person in cities and city sizes. Nations with lower productivity can nevertheless become wealthy as a whole by growing their cities larger, currently a worldwide phenomenon [46], whereas nations with high productivity can be rich even while having relatively small cities [47]. Spain or Poland already have large cities but could do well to increase their baseline productivity; Switzerland or The Netherlands could become even richer simply by growing their cities further.

This brings us to the issue of city size distributions for different European nations. This is usually summarized by Zipf's law (or rank-size rule) for the size distribution of cities [48,49]. Figure 7b shows the counter-cumulative normalized frequency distribution, $P(N \geq x)$, for the five

largest nations in the EU analysed above. Expressed in this way Zipf's law is simply $P(N \geq x) = N_{\min}/x$, where N_{\min} is the smallest city size in the dataset.

None of these five nations shows a city size distribution in good agreement with Zipf's law (figure 7). Although there is, as usual, some initial agreement for the smallest cities, France and the UK are very similar in that they have one enormous city and secondary large cities that are too small by Zipf law's expectation. Germany lacks large cities; Italy is very scattered. Spain is perhaps the urban system with the best agreement, but it is small and its two largest cities (Madrid and Barcelona) are too big compared with the Zipfian expectation. The inset in figure 7b shows the same distribution for all cities in Europe in the OECD–EU dataset (112 cities). We see that if we take Europe as an integrated urban system then its large cities are all too small and the Zipfian expectation fails to work at the pan-European level either. We conclude that Zipf's law, one of the oldest empirical regularities for cities, fails to give us any consistent expectation for the 'right' sizes of European cities, either at the national level or in the aggregate.

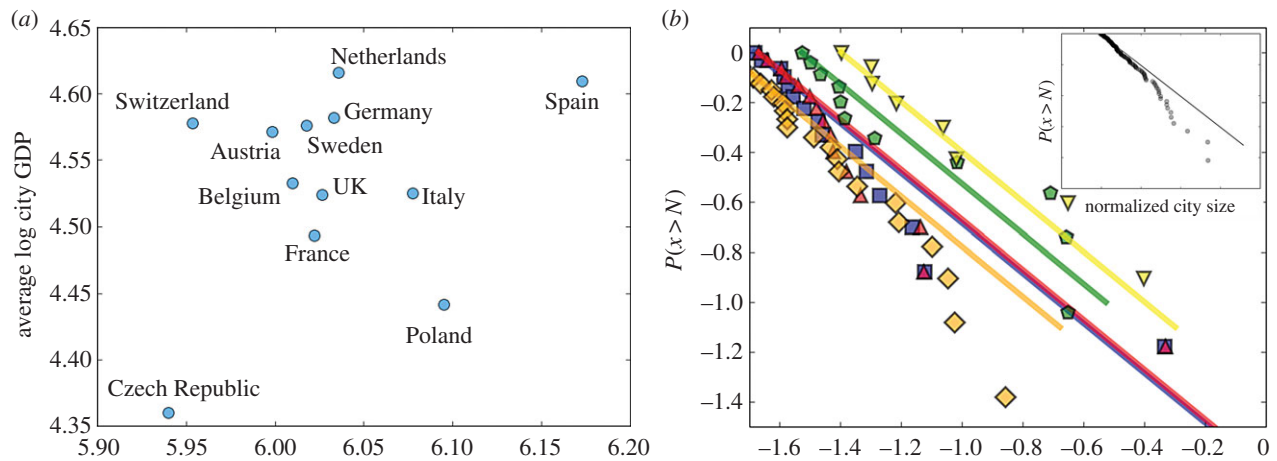


Figure 7. The average logarithm GDP and population size of cities in different national urban systems in Europe and their city size distributions. (a) The average logarithmic GDP and city size across cities in each nation. These are the values used to centre the data in figure 6. We see no correlation, as expected, but observe that different nations are characterized by different average city sizes and GDP magnitudes, see text for discussion. (b) The distribution of relative city sizes, $P(x \geq N)$, for France (blue), Germany (orange), Italy (green), Spain (yellow) and the UK (red) and for the EU, inset (112 cities $\geq 500\,000$ people). We observe that none of the largest Western European urban systems follows Zipf's law (solid lines): Germany lacks large enough cities; France and the UK are remarkably similar and are characterized by one very large city (Paris, London) and a set of secondary cities that are very small. When all the cities in the dataset are pooled together we find (inset) that Zipf's law (red line) is a poor descriptor of the size distribution of city sizes across the EU in the sense that Europe lacks large enough cities. This issue is explored in §2.4, where we derive the expected size of the largest city in Europe from Zipf's law (58 million!) and predict its GDP, land area and patent production from urban scaling. (Online version in colour.)

The disagreement between Zipf's law for the UK (or France) could lead us to conclude, for example, that London (Paris) is too large, too expensive and too destabilizing of other cities in the UK (France), a phenomenon that policy should actively address [50,51]. However, urban scaling tells us instead that London and Paris are not at all 'anomalous'. Shrinking London (or Paris), by moving population to other smaller cities, would make the UK (or France) poorer as a whole because London is the main way in which the nation explores the economic multiplicative effects of urban agglomeration. Such outcome would in any case probably be unacceptable and anathema to the spirit of the policies developed to improve the UK's urban system. Other scenarios that would make the UK (France) richer, would need to rely instead on moving population *up the urban hierarchy*, from smaller towns to mid-sized cities. Another, harder path, would attempt to replicate the Swiss or Dutch model and create a larger baseline productivity that allows all cities to do better without requiring demographic growth. For Germany, the path for larger wealth may have to do with growing its big cities larger, which, at least in the case of Berlin, remains a work in progress after the events of the twentieth century that curbed its early explosive growth.

We conclude this section with a set of *conjectures* for the implications of Zipf's law for Europe as a whole. Zipf's law emerges from urban systems that are in a type of demographic steady state (or 'equilibrium') in which all cities have the same average growth rate. Statistical fluctuations in such growth rates that are independent of city size then produce Zipf's rank-size rule [48,49]. Thus, an integrated pan-European urban system should eventually approximate such a distribution of city sizes. According to Zipf's law, and the empirical baselines of figure 7 (inset), we can calculate that the largest city in the EU should have a population of 58 million people, the second 29 million, the third about

19 million and so on. The fifth largest city would be roughly the size of today's Paris or London, with 11.5 million people. Using urban scaling, we can compute the GDP, urbanized area, patent production and many more quantities [15] for these hypothetical cities. The GDP of such largest city would be 6.5 times larger than that of Paris or London today, which corresponds to an increase of 30% *per capita* in these cities. A similar increase in density in these cities (decrease in urbanized area *per capita*) would also be expected, but this would result in a density for Paris of about 26 000 people km^{-2} , still much lower than earlier twentieth century densities and than some of the most exciting parts of Paris (presently, the 11th *arrondissement* is the densest in Paris with a population density of 40 000 people km^{-2}).

Creating such cities in Europe through the vigorous growth of Paris, London, Berlin, Madrid, Milan and others to such enormous sizes may certainly also have its drawbacks. Continuing urban system integration at the pan-European level is all but certain to grow these population centres disproportionately, further increasing inequalities between richer and poorer regions in Europe. But, at the same time, it would create a truly international culture in Europe, beyond today's heritage of older nationalisms and unleash massive technological and economic growth of the kind most European nations can currently only dream of.

3. Discussion

We have analysed the scaling properties of European MAs, defined by the most recent joint effort by the OECD–EU to create harmonized functional cities [26]. Arguably, these functional urban units constitute the best consistent definition of socio-economic cities in Europe constructed to date and allow for improved comparative analyses of urban properties throughout Europe and beyond. Using these

units, we find support for the quantitative predictions of urban scaling theory regarding scaling exponents, especially for France and the UK and in the aggregate of EU nations. This shows that urban scaling with the specific elasticities (exponents) discussed here is exhibited by urban systems whose constituent units are indeed functional urban units, even in Europe. Other plausible urban units of analysis (such as political cities or dense spatial clusters) are likely poorer approximations to cities as socio-economic units and should be expected to exhibit different elasticities or, possibly, no clear scaling at all [38,52]. By using units of analysis whose spatial delineations actually capture urban functionality, the British urban system and especially France, are shown to exhibit expected scaling behaviour, despite many historical and contemporary peculiarities [43,52]. There is also broad empirical agreement between the scaling patterns of European MAs and the properties of other large urban systems, such as the USA, China or Brazil [13–15]. The current OECD–EU dataset covers urban areas with populations larger than 500 000 people so it will be interesting to explore in future analyses smaller harmonized functional cities. Given the density and compactness of many European regions, these spatial units may in many cases be difficult to define unambiguously.

Despite the effort and the rationale involved in this most recent definition of functional cities in Europe, it should be expected that such definitions will continue to be improved in the future and we look forward to revisiting our empirical findings at such times. Urban scaling analysis provides a general simple expectation for many of the properties of a city in relation to its urban system and, as a consequence, it constitutes a means to identify places with exceptional properties, good and bad. Such deviations can be the result of true local exceptionality or of data issues. Our analysis flags a number of European MAs as exceptional, in their regional context and in Europe at large. The strongest deviations from scaling for GDP are observed for Naples, Italy and for Berlin, Germany. In both cases, these cities are either too large in population for their economic performance, or have economies that are too small for their populations. Berlin, and other former East Germany cities, having endured the ravages of World War II and the Cold War, still tend to underperform compared to their West German counterparts. But Berlin, given its history and recent policy interventions, clearly stands on its own as a particular case. It would be interesting to analyse the sensitivity of the economic performance for these cities versus their spatial definitions further and to follow their temporal evolution closely. Similarly, and possibly related, the urbanized area of Naples appears too small and that of Madrid, Spain too large. In this way, urban scaling analysis, can be used to point the way towards better quantitative and systematic understanding of the exceptionality of specific places and for better identifying their specific contingencies and histories from a quantitative perspective. The results presented here should be revisited as data for smaller European MAs (population < 500 000) become available as these urban areas, of which there are many, interact with and affect the performance of the larger MAs. Expanding the available data to include smaller metropolitan regions would of course increase the number of urban observations within each country and for the EU as a whole, a statistically desirable situation.

On the strength of the results presented here and those from previous studies of other contemporary and older urban

systems, we can conclude that urban scaling is a stronger statistical regularity for functional cities than the rank-size distribution of city size, also known as Zipf's law. Different European nations are characterized by very different rank-size distributions, with example, France or the UK show strong primacy (as Paris and London are four to five times larger than the second largest city), whereas in Germany, Spain or Italy the opposite is true and several large cities coexist. Despite these well-known and very variable patterns in the size distribution of cities, agglomeration effects and the resulting scaling relations persist with greater regularity in each system and especially in the analysis that pools all nations in Europe together. From the perspective of urban scaling, London and Paris are not exceptional cities at all. Rather, their properties are just what one should expect for a British or French city of their population size. Thus, it will be interesting to continue to develop urban theory that brings together the theoretical insights behind scaling and agglomeration effects with those that predict the size distribution of cities.

Today, Europe remains a less urbanized continent than North America or developed Asia, with an overall urbanization rate of about 70%. Paris and London are growing slowly, with annual growth rates of 0.68% and 1.18%, respectively, typical of most EU cities. Madrid is Western Europe's fastest growing large city with an annual growth rate of 1.8% [26]. At this pace, London would double its population in 61 years, Paris in 106, and Madrid in 40 years. As a thought experiment, we extrapolated the expectations of Zipf's law for the size of the largest cities in the EU to predict a city with population above 50 million people and a number of other very large pan-European megacities. Using urban scaling theory, we predicted (conservatively, in the absence of additional economic growth) that such a city would be an economic colossus, with a GDP and invention rate *per capita* 30% larger than those of Paris and London today. Thus, the rise of pan-European megacities would create tremendous magnification effects to wealth creation and technological invention that would help keep Europe on par with other large and fast developing nations, such as the USA, Japan and future developed versions of China and India. Such massive transformations of Europe's urban system would, however, also severely exacerbate regional inequalities by further amplifying the wealth, technology and organizational sophistication of the richest areas of Europe today.

Authors' contributions. L.M.A.B. conceived and designed the study, carried out the statistical analysis and drafted the manuscript; J.L. conceived the study and helped draft the manuscript. All authors gave final approval for publication.

Competing interests. We declare we have no competing interests.

Funding. This work was partially funded by the MacArthur Foundation (grant 13-105749-000-USP) and the Army Research Office Minerva program (Grant W911NF1210097) (L.M.A.B.). We also thank the Arizona State University/Santa Fe Institute Center for Biosocial Complex Systems for support (L.M.A.B. and J.L.).

Acknowledgements. We thank Deborah Strumsky for discussions and Daniel Sanchez Serra for clarifications of the OECD–EU definitions of harmonized functional cities.

Endnotes

¹For historical definitions of MSAs, see <http://www.census.gov/population/metro/data/pastmetro.html>.

²See https://en.wikipedia.org/wiki/Nomenclature_of_Territorial_Units_for_Statistics.

³For a detailed discussion of how the EU and the OECD have delineated metropolitan areas, see [http://www.oecd.org/gov/regional-policy/Definition-of-Functional-Urban-Areas-for-the-OECD-](http://www.oecd.org/gov/regional-policy/Definition-of-Functional-Urban-Areas-for-the-OECD-metropolitan-database.pdf)

[metropolitan-database.pdf](http://www.oecd.org/gov/regional-policy/Definition-of-Functional-Urban-Areas-for-the-OECD-metropolitan-database.pdf). For maps and all data, see <http://measuringurban.oecd.org>.

⁴Note that $Y(N) = Y_0 N^{1+\delta} = Y_0 N e^{\delta \ln N} = Y_0 N (1 + \delta \ln N + 1/2(\delta \ln N)^2 + \dots) \simeq Y_0 N \ln N / e$, for small $\delta \ln N$. So if $\ln N < 1/\delta$, see below, these functions are the same analytically.

References

- Braudel F. 1992 *Civilization and capitalism, 15th–18th century*. Berkeley, CA: University of California Press.
- De Vries JD, Vries J. 2013 *European urbanisation: 1500–1800*. No. B. London, UK: Routledge.
- Bairoch P. 1991 *Cities and economic development: from the dawn of history to the present*. Chicago, IL: University of Chicago Press.
- Hall P. 1998 *Cities in civilization*. New York, NY: Pantheon Books.
- Mumford L. 1989 *The city in history: its origins, its transformations, and its prospects*. San Diego, CA: Harcourt.
- Henderson JV. 1991 *Urban development: theory, fact, and illusion*. New York, NY: Oxford University Press.
- Fujita M, Krugman P, Venables AJ. 2001 *The spatial economy: cities, regions, and international trade*. Cambridge, MA: The MIT Press.
- Glaeser EL. 2012 *Triumph of the city: how our greatest invention makes us richer, smarter, greener, healthier, and happier*. New York, NY: Penguin Books.
- Isard W. 1972 *Location and space-economy: a general theory relating to industrial location, market areas, land use, trade, and urban structure*. Cambridge, MA: MIT Press.
- Batty M. 2008 The size, scale, and shape of cities. *Science* **319**, 769–771. (doi:10.1126/science.1151419)
- Storper M. 2013 *Keys to the city: how economics, institutions, social interactions, and politics shape development*. Princeton, NJ: Princeton University Press.
- Kennedy CM. 2011 *The evolution of great world cities: urban wealth and economic growth*. Toronto, Canada: University of Toronto Press.
- Bettencourt LMA, Lobo J, Helbing D, Khnert C, West GB. 2007 Growth, innovation, scaling, and the pace of life in cities. *Proc. Natl Acad. Sci. USA* **104**, 7301–7306. (doi:10.1073/pnas.0610172104)
- Bettencourt LMA, Lobo J, Youn H. 2013 The hypothesis of urban scaling: formalization, implications and challenges. (<http://arxiv.org/abs/1301.5919>).
- Bettencourt LMA. 2013 The origins of scaling in cities. *Science* **340**, 1438–1441. (doi:10.1126/science.1235823)
- Duranton G, Puga D. 2004 Micro-foundations of urban agglomeration economies. In *Handbook of regional and urban economics*, vol. 4 (eds JV Henderson, J-F Thisse), pp. 2063–2117. Amsterdam, The Netherlands: Elsevier.
- Barenblatt GI. 1996 *Scaling, self-similarity, and intermediate asymptotics*. Cambridge, UK: Cambridge University Press.
- Bettencourt LMA, Lobo J, Strumsky D, West GB. 2010 Urban scaling and its deviations: revealing the structure of wealth, innovation and crime across cities. *PLoS ONE* **5**, e13541. (doi:10.1371/journal.pone.0013541)
- Pred A. 1977 *City systems in advanced economies: past growth, present processes, and future development options*. New York, NY: Wiley.
- Glaeser EL, Scheinkman J, Shleifer A. 1995 Economic growth in a cross-section of cities. *J. Monetary Econ.* **36**, 117–143. (doi:10.1016/0304-3932(95) 01206-2)
- Ciccone A. 2002 Agglomeration effects in Europe. *Eur. Econ. Rev.* **2**, 213–227. (doi:10.1016/S0014-2921(00)00099-4)
- Rice P, Venables AJ, Patacchini E. 2006 Spatial determinants of productivity: analysis for the regions of Great Britain. *Reg. Sci. Urban Econ.* **36**, 727–752. (doi:10.1016/j.regsciurbeco.2006.03.006)
- Patacchini E. 2008 Local analysis of economic disparities in Italy: a spatial statistics approach. *Stat. Methods Appl.* **17**, 85–112. (doi:10.1007/s10260-007-0054-8)
- van Raan AFJ, van der Meulen G, Goedhart W. 2015 Urban scaling of cities in the Netherlands. (<http://arxiv.org/abs/1503.04795>)
- Paulus F, Pumain D, Vacchiani-Marcuzzo C, Lobo J. 2006 An evolutionary theory for interpreting urban scaling laws. See <https://halshs.archives-ouvertes.fr/CSS/halshs-00138790v1>.
- OECD (ed.). 2012 *Redefining 'urban': a new way to measure metropolitan areas*. Paris, France: OECD.
- Woude AMVD, Hayami A, De Vries J, International Union for the Scientific Study of Population (eds). 1990 *Urbanization in history: a process of dynamic interactions*. International studies in demography, Oxford, UK: Oxford University Press.
- Pan W, Ghoshal G, Krumme C, Cebrian M, Pentland A. 2013 Urban characteristics attributable to density-driven tie formation. *Nat. Commun.* **4**, 1961. (doi:10.1038/ncomms2951)
- Sim A, Yaliraki SN, Barahona M, Stumpf MPH. 2015 Great cities look small. *J. R. Soc. Interface* **12**, 20150315. (doi:10.1098/rsif.2015.0315)
- Ortman SG, Cabaniss AHF, Sturm JO, Bettencourt LMA. 2014 The pre-history of urban scaling. *PLoS ONE* **9**, e87902. (doi:10.1371/journal.pone.0087902)
- Ortman SG, Cabaniss AHF, Sturm JO, Bettencourt LMA. 2015 Settlement scaling and increasing returns in an ancient society. *Sci. Adv.* **1**, e1400066. (doi:10.1126/sciadv.1400066)
- Gomez-Lievano A, Youn H, Bettencourt LMA. 2012 The statistics of urban scaling and their connection to Zipf's law. *PLoS ONE* **7**, e40393. (doi:10.1371/journal.pone.0040393)
- Alves LGA, Mendes RS, Lenzi EK, Ribeiro HV. 2014 Scale-adjusted metrics for predicting the evolution of urban indicators and quantifying the performance of cities. (<http://arxiv.org/abs/1507.05204>)
- Rosenthal SS, Strange WC. 2004 Evidence on the nature and sources of agglomeration economies. In *Handbook of regional and urban economics*, vol. 4 (eds JV Henderson, J-F Thisse), pp. 2119–2171. Amsterdam, The Netherlands: Elsevier.
- Alonso W. 1977 *Location and land use: toward a general theory of land rent*. Cambridge, MA: Harvard University Press.
- Black D, Henderson V. 1999 A theory of urban growth. *J. Polit. Econ.* **107**, 252–284. (doi:10.1086/250060)
- Shapiro C, Varian H. 1999 *Information rules: a strategic guide to the network economy*. Cambridge, MA: Harvard Business School.
- Louf R, Barthelemy M. 2014 Scaling: lost in the smog. *Environ. Plann. B Plann. Des.* **41**, 767–769. (doi:10.1068/b4105c)
- Sjoberg G. 1965 *The preindustrial city: past and present*. New York, NY: Free Press.
- Gies J, Gies F. 1969 *Life in a medieval city*, 1st edn. New York, NY: HarperPerennial.
- Bettencourt LM, Lobo J, Strumsky D. 2007 Invention in the city: increasing returns to patenting as a scaling function of metropolitan size. *Res. Policy* **36**, 107–120. (doi:10.1016/j.respol.2006.09.026)
- Nicholas D. 1997 *The later medieval city, 1300–1500. A history of urban society in Europe*. London, UK: Longman.
- Arcaute E, Hatna E, Ferguson P, Youn H, Johansson A, Batty M. 2014 Constructing cities, deconstructing scaling laws. *J. R. Soc. Interface* **12**, 20140745. (doi:10.1098/rsif.2014.0745)
- Clifton N. 2008 The 'creative class' in the UK: an initial analysis. *Geografiska Annaler, Series B: Hum. Geography* **90**, 63–82. (doi:10.1111/j.1468-0467.2008.00276.x)
- Plöger J. 2007 Bremen city report. *Centre for Analysis of Social Exclusion CASereport* 39. See [http://eprints.lse.ac.uk/3623/1/Bremen_city_report_\(final\).pdf](http://eprints.lse.ac.uk/3623/1/Bremen_city_report_(final).pdf).
- Bloom DE, Canning D, Fink G. 2008 Urbanization and the wealth of nations. *Science* **319**, 772–775. (doi:10.1126/science.1153057)
- McKinsey & Company. 2011 Urban world: mapping the economic power of cities. See http://www.mckinsey.com/insights/urbanization/urban_world.

48. Zipf GK. 2012 *Human behavior and the principle of least effort: an introduction to human ecology*. Mansfield Centre, CT: Martino.
49. Gabaix X. 1999 Zipf's law for cities: an explanation. *Q. J. Econ.* **114**, 739–767. (doi:10.1162/003355399556133)
50. Champion T, Townsend A. 2013 Great Britain's second-order city regions in recessions. *Environ. Plann. A* **45**, 362–382. (doi:10.1068/a45100)
51. Champion T, Coombes M, Gordon I. 2014 How far do England's second-order cities emulate London as human-capital escalators? England's second-order cities as human-capital escalators? *Popul. Space Place* **20**, 421–433. (doi:10.1002/psp.1806)
52. Cottineau C, Hatna E, Arcaute E, Batty M. 2014 Paradoxical interpretations of urban scaling laws. (<http://arxiv.org/abs/1507.07878>)